Workshop on Item Response Theory Models in Political Science

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Applications in Political Science I

IRT models have become very influential in political science.

- Measure Political Knowledge

Tsai, T. H., & Lin, C. C. (2017). Modeling Guessing Components in the Measurement of Political Knowledge. Political Analysis, 25(4), 483-504.

- Democracy as a latent construct

Treier, S., & Jackman, S. (2008). Democracy as a latent variable. American Journal of Political Science, 52(1), 201–217.

- Statistical Models of Roll Call Voting

Applications in Political Science II

Clinton, J., Jackman, S., & Rivers, D. (2004). The statistical analysis of roll call data. American Political Science Review, 98(2), 355-370.

- Public Opinion

Treier, S., & Hillygus, D. S. (2009). The nature of political ideology in the contemporary electorate. Public Opinion Quarterly, 73(4), 679-703.

Zhou, X. (2019). Hierarchical Item Response Models for Analyzing Public Opinion. Political Analysis.

- Dynamic Area Estimates

Caughey, D., & Warshaw, C. (2016). The Dynamics of State Policy Liberalism, 1936–2014. American Journal of Political Science, 60(4), 899–913.

Applications in Political Science III

- Ideology of Judges

Martin, A., & Quinn K. (2002). Dynamic Ideal Point Estimation via Markov Chain Monte Carlo for the U.S. Supreme Court, 1953-1999. Political Analysis, 10, 134-153.

- Social Media Networks

Barber, P. (2015). Birds of the same feather tweet together: Bayesian ideal point estimation using Twitter data. Political Analysis, 23(1), 76-91.

Workshop aims

- Introduce basics IRT Models
- Application to the statistical analysis of roll call votes
- Learn about R-software to estimate IRT Models

Item Response Theory

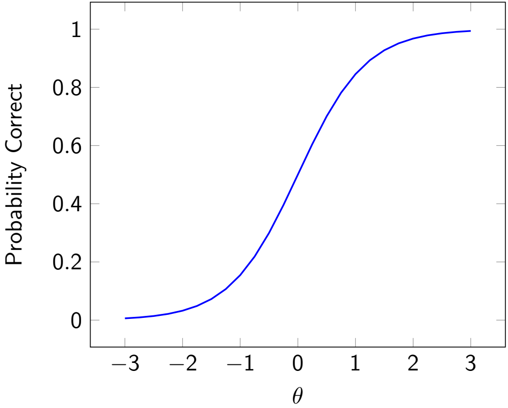
Item response theory (IRT) models show the relationship between the ability or trait measured by the instrument and an item response.

- The item response might be *dichotomous* (Two catgories) or *polytomous* (e.g. Likert scales)
- The construct measured might be proficiency or aptitude, or an attitude or belief

Item Response Theory

- Consider the trait θ, which could be political knowledge
- We have a binary item, for example, a factual question about politics. E.g. who is the current foreign minister?
- We code $Y_i = 1$ if the question is answered correctly and 0 otherwise
- We relate $P(Y_i = 1)$ to θ, assuming a monotonic, increasing function

Rasch Model



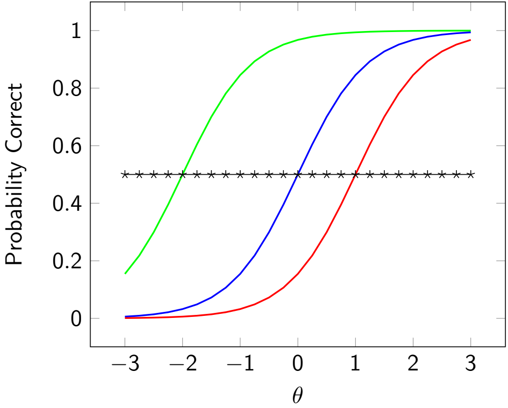
Rasch Model II

$$P(y_i = 1|\theta; \beta_i) = \frac{\exp(D \cdot (\theta - \alpha_i))}{1 + \exp(D \cdot (\theta - \beta_i))}$$

where

- θ is ability/trait
- β_i is the item difficulty
- D is a scaling factor; for ≈ 1.7 the logistic model behaves like a standard normal one

Rasch Model II



Red item is most difficult: highest θ to achieve $P = 0.5$; green item is least difficult

Alternative Notation

The Rasch model is sometimes also presented in terms of the normal ogive:

$$P(y_i = 1|\theta; \beta_i) = \Phi(\theta - \beta_i)$$

Local Independence

We assume local independence:

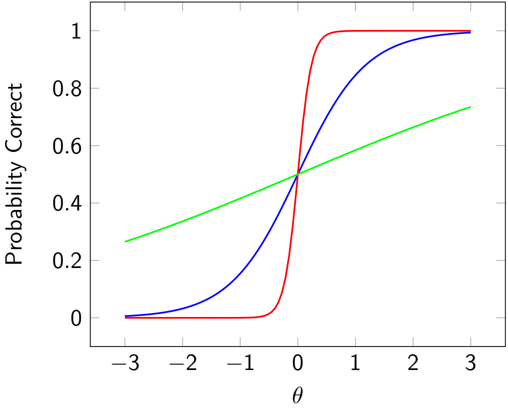
> *When we hold θ constant, then the responses to any pair of items are statistically independent*

Two-Parameter IRT

$$P(y_i = 1|\theta; \alpha_i, \beta_i) = \frac{\exp(\alpha_i \cdot D \cdot (\theta - \beta_i))}{1 + \exp(\alpha_i \cdot D \cdot (\theta - \beta_i))}$$

$$\alpha_i = \text{Item Discrimination Parameter}$$

Two-Parameter IRT



Red item has greatest discrimination; green item has the least

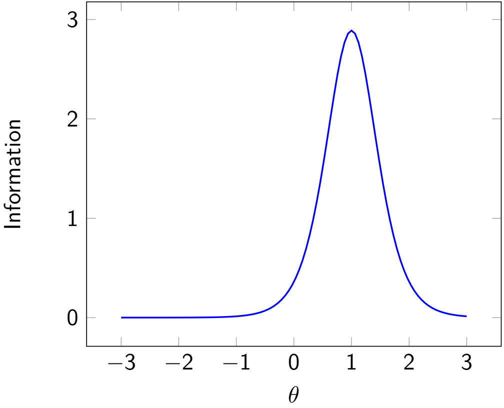
Item Information

The expression

$$
\begin{aligned}
I_i &= \frac{\left(\frac{dP(y_i=1|\theta;\alpha_i,\beta_i)}{d\theta}\right)^2}{P(y_i=1|\theta;\alpha_i,\beta_i)\cdot P(y_i=0|\theta;\alpha_i,\beta_i)} \\
&= \frac{D^2\alpha_i^2}{\exp(D\cdot\alpha_i\cdot(\theta-\beta_i))\cdot(1+\exp(-D\cdot\alpha_i\cdot(\theta-\beta_i)))^2}
\end{aligned}
$$

gives the amount of information an item provides at a given value of θ.

Item Information Functions

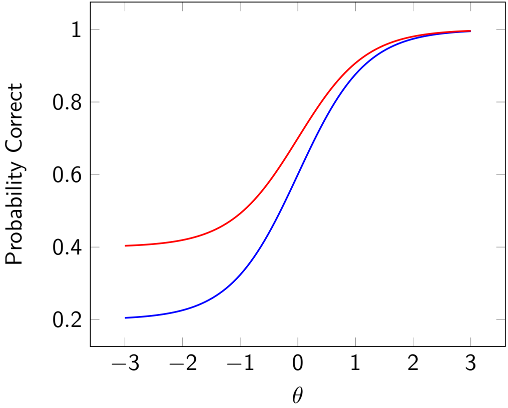


$$\alpha = 2;\ \beta = 1$$

Three-Parameter IRT

$$
\begin{aligned}
P(y_i = 1|\theta; \alpha_i, \beta_i, c_i) &= c_i + (1 - c_i)\frac{\exp(\alpha_i \cdot D \cdot (\theta - \beta_i))}{1 + \exp(\alpha_i \cdot D \cdot (\theta - \beta_i))} \\
\alpha_i &= \text{Item Discrimination Parameter}
\end{aligned}
$$

Three-Parameter IRT



red item has a higher c_i parameter

Inference Estimands

1. N latent scores θ
2. K item difficulties
3. K item discrimination parameters

Likelihood-Based Inference

- Assume a set of binary items
- For a respondent, q, we obtain a pattern of 1s and 0s
- The 1s appear with probabilities $p_{qi} = P(y_{qi} = 1|\theta_q; \alpha_i, \beta_i)$
- The 0s appear with probabilities $n_{qi} = 1 - P(y_{qi} = 1|\theta_q; \alpha_i, \beta_i)$
- For a specific individual, the likelihood is now

$$L_q = \prod_{i=1}^{K} p_{qi}^{y_{qi}} n_{qi}^{1-y_{qi}}$$

Likelihood-Based Inference

Across all respondents, we have

$$L = \prod_{q=1}^{N} \prod_{i=1}^{K} p_{qi}^{y_{qi}} n_{qi}^{1-y_{qi}}$$

We optimize this with respect to the θs, αs, and βs

Approaches

- Some software estimates all parameters simultaneously.
- Other software uses marginal maximum likelihood:
  - First, integrate out θ from the likelihood and estimate the item parameters
  - Next, estimate the θs given the item parameters

Exercise: An IRT model for Political Ideology

The CCES asks respondents how they would have voted on a set of parliamentary bills. For example: Would troops be withdraw from Iraq in the next 180 days? Yes or No. The answers can be used as a direct measure of respondents ideology.

- Define a item response Model for the answering behavior using a 2PL formulation. (For a list of all items see the next slide)
- What item response curves would you expect? Draw two examples in a graph

Exercise: An IRT model for Political Ideology

ID	CCES	Short Description	CCES Wording
rv1	CC316a	Withdraw Troops Iraq	Withdraw Troops from Iraq within 180 days
rv2	CC316b	Increase Minimum Wage	Increase Minimum Wage from $5.15 to $7.25
rv3	CC316c	Stem Cell Research	Allow federal funding of embryonic stem cell research
rv4	CC316d	Foreign Intelligence Surveillance Act	Allow U. S. spy agencies to eavesdrop on overseas terrorist suspects without first getting a court order
rv5	CC316e	Children's Health Insurance	Fund a $20 billion program to provide health insurance for children in families earning less that $43,000
rv6	CC316f	Anti Gay Marriage Amendment	Constitutional Amendment banning Gay Marriage
rv7	CC316g	Federal Assistance for Housing Crisis	Federal assistance for homeowners facing foreclosure and large lending institutions at risk of failing
rv8	CC316h	Extend NAFTA	Extend the North American Free trade Agreement (NAFTA) to include Peru and Columbia
rv9	CC316i	Bank Bailout	Governments $700 Billion Bank Bailout Plan

Table 1: CCES 2008 Roll Call Vote Questions

Figure 1: Issue Items

Application: The statistical analysis of roll call data

The aim of the session is to apply Item Response Theory to the analysis of Roll Call Votes to estimate latent ideologies of legislatures.

We follow Clinton et. al and derive IRT models from spatial model of voting

And estimate the model for one session of the U.S. senate.

Spatial Model of Roll Call Voting

- n legislators and m different roll call votes
- Each roll call $j = 1, \ldots, m$ represents legislators $i \in 1, \ldots, n$ with choice between "Yeah" position ζ_j and "Nay" position ψ_j
- Both are defined in $\mathcal{R}^d$ multiple dimensions
- $y_{ij} = 1$ if legislator i votes "Yeah" on proposal j, and $y_{ij} = 0$ if "Nay"

Spatial Model of Roll Call Voting I

- Quadratic Loss from legislators ideal $\boldsymbol{x}_i$ to the proposals:

$$U_i(\zeta_j) = -(\boldsymbol{x}_i - \zeta_j)^2 + \eta_{ij} \quad (1)$$
$$U_i(\psi_j) = -(\boldsymbol{x}_i - \psi_j)^2 + \nu_{ij} \quad (2)$$

- Utility maximization implies that

$$y_{ij} = 1 \quad \text{if} \quad U_i(\zeta_j) > U_i(\psi_j) \quad (3)$$
$$y_{ij} = 0 \quad \text{otherwise} \quad (4)$$

Spatial Model of Roll Call Voting II

- Assume that normal distributed errors with $E(\eta_{ij}) = E(\nu_{ij})$ and $Var(\eta_{ij} - \nu_{ij}) = 1$
- We can derive an 2PL item response formulation:

$$
\begin{aligned}
P(y_{ij} = 1) &= P(U_i(\zeta_j) > U_i(\psi_j)) && (5)\\
&= P(\nu_{ij} - \eta_{ij} > (\boldsymbol{x}_i - \psi_j)^2 - (\boldsymbol{x}_i - \zeta_j)^2) && (6)\\
&= P(\nu_{ij} - \eta_{ij} < 2(\zeta_j - \psi_j)'\boldsymbol{x}_i + \psi_j'\psi_j - \zeta_j'\zeta_j && (7)\\
&= \Phi(\boldsymbol{\beta}_j'\boldsymbol{x}_i - \alpha_j) && (8)
\end{aligned}
$$

- where $\boldsymbol{\beta}_j = 2(\zeta_j - \psi_j)$, $\alpha_j = \psi_j'\psi_j - \zeta_j'\zeta_j$, and $\Phi(\cdot)$ CDF of the standard normal distribution.

Spatial Model of Roll Call Voting III

- In political science ideal point models are often estimated using Bayesian methods, which has the advantage that ideal points and item parameters are estimated simultaneously.
- For Bayesian inference: it requires prior distributions for the parameters of the model. Often use vague prior information about the parameters: $\beta_j \sim N(0, \sigma^2_{\beta_j})$ and $\alpha_j \sim N(0, \sigma^2_{\alpha_j})$ and $x_i \sim N(0, 1)$.
- In addition often a set of legislators ideal points are fixed to identify the model.

Spatial Model of Roll Call Voting IV

- This means we can estimate a 2PL model on the Roll-Call vote matrix and interpret the results in terms of spatial voting model about proposals
- There are different ways to estimate the model.
- Bayesian estimation using MCMC, Clinton et al. and R-implementation ideal. Time-consuming but complete posterior distribution
- Expectation Maximization Algorithm to approximate posterior, Imai et al., R-implementation emIRT. Fast, requires bootstrap to get standard errors

R-Code

Application of Model to US Senate